Webwork 4.4

① ⓐ $\vec{x} = \begin{bmatrix}-1\\-1\\4\end{bmatrix}$, $E = \left\{ \begin{bmatrix}1\\6\\2\end{bmatrix}, \begin{bmatrix}0\\1\\7\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$

$\begin{bmatrix}-1\\-1\\4\end{bmatrix} = a\begin{bmatrix}1\\6\\2\end{bmatrix} + b\begin{bmatrix}0\\1\\-7\end{bmatrix} + c\begin{bmatrix}0\\0\\1\end{bmatrix}$

$a = -1$

$6a + b = -1$

$-6 + b = -1$

$b = 5$

$2a - 7b + c = 4$

$-2 - 35 + c = 4$

$c = 41$

coordinate vector

$[x]_E = \begin{bmatrix}-1\\5\\41\end{bmatrix}$

$0 = 4 - 33$

ⓑ $F_1 = \left\{ \begin{bmatrix}3\\3\end{bmatrix}, \begin{bmatrix}-1\\5\end{bmatrix} \right\}$

$F_2 = \left\{ \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}3\\2\end{bmatrix} \right\}$

Transition matrix $P_{F_2 \leftarrow F_1}$ such that $P_{F_2 \leftarrow F_1}[x]_{F_1}$ for all $x$ in $\mathbb{R}^2$

$\begin{bmatrix}3\\3\end{bmatrix} = p_1\begin{bmatrix}2\\1\end{bmatrix} + p_2\begin{bmatrix}3\\2\end{bmatrix}$

$2p_1 + 3p_2 = 3$

$2p_1 + 4p_2 = 6$

$-p_2 = -3$

$2p_1 + 3p_2 - p_1 - 2p_2 = 0$

$p_1 + p_2 = 0$

$\cancel{p_1 \times p_2}$

$p_2 = 3$

$2p_1 = 3 - 9$

$p_1 = 3$

$\begin{bmatrix}3\\3\end{bmatrix} = 3\begin{bmatrix}2\\1\end{bmatrix} + 3\begin{bmatrix}3\\2\end{bmatrix}$

$\begin{bmatrix}-1\\5\end{bmatrix} = q_1\begin{bmatrix}2\\1\end{bmatrix} + q_2\begin{bmatrix}3\\2\end{bmatrix} \Rightarrow 2q_1 + 3q_2 = -1$

$2q_1 + 4q_2 = 10$

$-q_2 = -11$

$q_2 = 11 \Rightarrow q_1 = -17$

$2q_1 + 33 = -1$

$q_1 = \frac{-34}{2} = -17$

$\begin{bmatrix}-1\\5\end{bmatrix} = (-17)\begin{bmatrix}2\\1\end{bmatrix} + (11)\begin{bmatrix}3\\2\end{bmatrix}$

Transition matrix $P = \begin{bmatrix}3 & -17\\3 & 11\end{bmatrix}$

② Basis of $\mathbb{R}^2$ is $B = \left\{ \begin{bmatrix}-3\\0\end{bmatrix}^T, \begin{bmatrix}-7\\-5\end{bmatrix}^T \right\}$

Find vector $\vec{x}$ such that the coordinate vector relative to basis $B$ $[x]_B$ is $\begin{bmatrix}5\\1\end{bmatrix}$

$\vec{x} = 5\begin{bmatrix}-3\\0\end{bmatrix} + 1\begin{bmatrix}-7\\-5\end{bmatrix} = \begin{bmatrix}-15-7\\0-5\end{bmatrix} = \begin{bmatrix}-22\\-5\end{bmatrix}$

③ $\vec{x} = \begin{bmatrix}-5\\-2\\-4\end{bmatrix}$, basis $B = \left\{ \begin{bmatrix}1\\8\\8\end{bmatrix}, \begin{bmatrix}0\\1\\-5\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix} \right\}$, find $[x]_B$ (coordinate vector)

$c_1\begin{bmatrix}1\\8\\8\end{bmatrix} + c_2\begin{bmatrix}0\\1\\-5\end{bmatrix} + c_3\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-5\\-2\\-4\end{bmatrix} \Rightarrow c_1 = -5,\ c_2 = -2 - 8(-5) = 38$

$c_3 = -4 - 8(-5) - 5(38) = -154$

$+40 - 190 + c_3 = -4$

$c_3 = -4 + 230 = 226$

④